Practice problems

Greedy

Collaboration level 0 (no restrictions). Open notes.

1. **Pharmacist problem:** A pharmacist has W pills and n empty bottles. Bottle i can hold p_i pills and has an associated cost c_i. Given W, $\{p_1, p_2, ..., p_n\}$ and $\{c_1, c_2, ..., c_n\}$, you want to store all pills using a set of bottles in such a way that the total cost of the bottles is minimized. Note: If you use a bottle you have to pay for its cost no matter if you fill it to capacity or not.

   Find the minimum cost for storing the W pills using the bottles.

   (a) Explain how the problem has optimal substructure.

   Answer: Consider an optimal solution O, and consider one of the bottles in it. Let's say this is bottle k, and it holds p_k pills. Then we know that the remaining bottles in O must be the optimal way to store .....

   (b) Define a subproblem and give pseudocode for a recursive function to compute it.

   (c) Extend your recursive pseudocode above to a recursive dynamic programming algorithm with memoization and analyze its running time.

2. **Greedy pharmacist?** Someone proposes the following greedy strategy to solve the pharmacist problem (above): Pick the bottle with the smallest cost-per-pill, and recurse on the remaining pills with the remaining bottles. Show that this greedy strategy is not correct by giving a counterexample.

3. **A different pharmacist problem:** A pharmacist has W pills and n empty bottles, where all bottles cost the same and bottle i can hold p_i pills. Find the minimum cost for storing the W pills using the bottles.